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# Ka-Band metalens antenna empowered by physics-assisted particle swarm optimization (PA-PSO) algorithm

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**Section 1: The derivation of extrema condition based on variation method of calculus**

The extrema condition of  $I(\varphi_m(r))$  can be derived by the variation method of calculus<sup>37</sup>, i.e.,

$$I(\varphi_m(r) + \delta(r)) - I(\varphi_m(r)) = 0. \tag{S1}$$

where  $\varphi_m(r) + \delta(r)$  is the variation of  $\varphi_m(r)$ , i.e.,  $\delta(r) \rightarrow 0$ .

According to Eqs. (4) and (5),

$$E(\varphi_m(r) + \delta(r)) = \int_0^R r e^{(\varphi_m(r) + \delta(r))j} \varepsilon(r) dr. \tag{S2}$$

Based on Taylor expansion

$$e^{\delta(r)j} = 1 + \delta(r)j, \text{ when } \delta(r) \rightarrow 0. \tag{S3}$$

Then Eq. (S2) can be written as,

$$E(\varphi_m(r) + \delta(r)) = E(\varphi_m(r)) + \int_0^R r e^{(\varphi_m(r))j} \varepsilon(r) \delta(r) j dr. \tag{S4}$$

Substituting Eqs. (S4) and (5) to Eq. (S1) and neglecting high order terms of  $\delta(r)$ ,

$$2\text{Re} \left[ E(\varphi_m(r))^* \int_0^R r e^{(\varphi_m(r))j} \varepsilon(r) \delta(r) j dr \right] = 0. \tag{S5}$$

According to Eq. (4),

$$2\text{Re} \left[ \left( \int_0^R r e^{(\varphi_m(r))j} \varepsilon(r) dr \right)^* \int_0^R r e^{(\varphi_m(r))j} \varepsilon(r) \delta(r) j dr \right] = 0. \tag{S6}$$

Then  $\left( \int_0^R r e^{(\varphi_m(r))j} \varepsilon(r) \delta(r) j dr \right)^* \int_0^R r e^{(\varphi_m(r))j} \varepsilon(r) \delta(r) j dr$  is either a pure imaginary number or zero. Therefore, the extrema condition of radiation intensity  $I$  can be satisfied when  $e^{(\varphi_m(r))j} \varepsilon(r)$  has a constant phase regardless of  $r$ , i.e.,  $\varphi_m(r) + \text{phase}(\varepsilon(r)) = \text{constant}$ , or  $E(\varphi_m(r)) = 0$ .

**Section 2: The flow diagram of PSO and PA-PSO algorithm**

The design of the metalens is based on the Particle Swarm Optimization (PSO) algorithm. The optimization algorithm for the metalens is as follows:

- 1、 Determine the size of the metalens lens and divide it into  $L$  rings based on the unit cell periodicity.
- 2、 Determine the coordinates of the unit cells in the  $i$ th ring.
- 3、 Initialize  $u^{\wedge}o = \text{rand}(1, L) \cdot 2\pi = ("u\_1", "u\_2", \dots, "u\_L")$  as  $N$  random values within the range of 0 to  $2\pi$ , where  $u^{\wedge}o$  represents the initial random values of phase for the  $i$ th ring.
- 4、 Assign particle values to each unit cell in the  $i$ th ring.
- 5、 Calculate the phase of the superposed electric field using the Kirchhoff diffraction formula. Perform this calculation for each population and repeat  $N$  times. Select the population with phase values closest to those of the first ring and keep them.
- 6、 For the remaining values, update using the following equations,

$$v^{\wedge}(g+1) = v^{\wedge}g + c\_1 \cdot \text{rand}(1) \cdot (p\_best - u^{\wedge}g) + c\_2 \cdot \text{rand}(1) \cdot (g\_best - u^{\wedge}g)$$

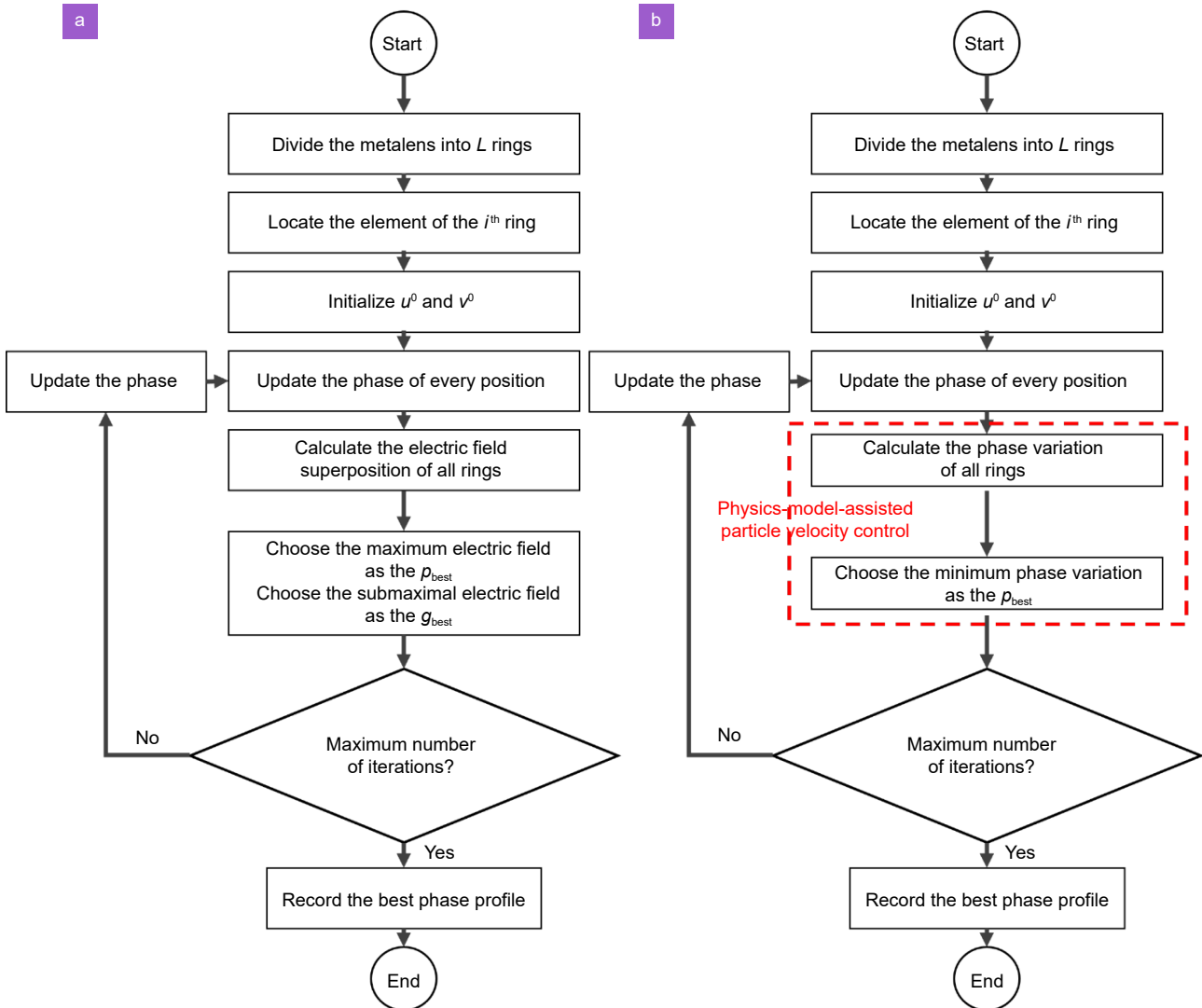
$$u^{\wedge}(g+1) = u^{\wedge}g + v^{\wedge}(g+1)$$

- 7、 Repeat the iterations until the phase of the  $i$ th ring matches the phase of the first ring.
- 8、 Calculate the phases for all rings using steps 2 to 7.

Shown in Fig. S1, the detailed procedure of the Physics-Assisted Particle Swarm Optimization (PA-PSO) algorithm is as follows:

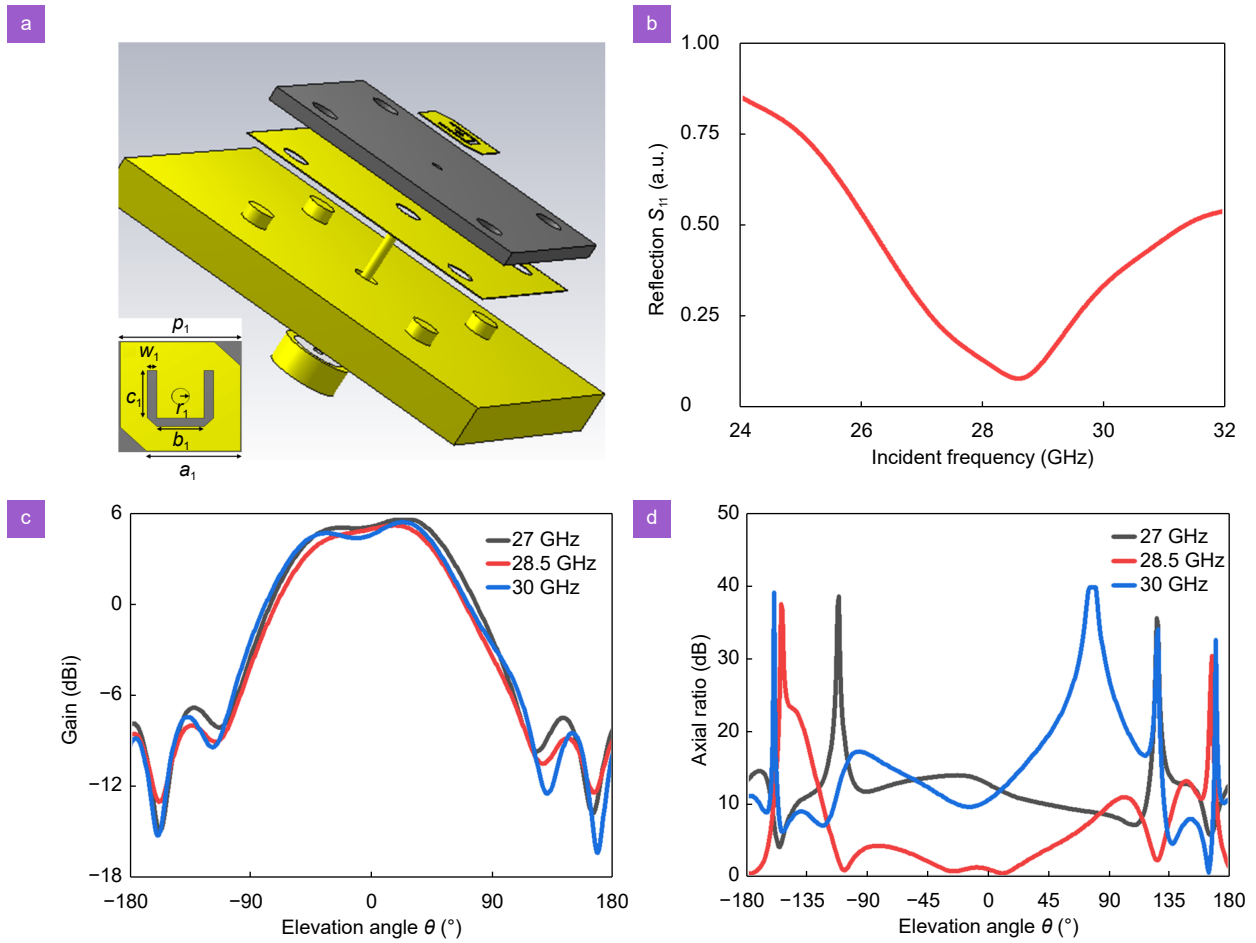
1. Dividing the lens into  $L$  rings: Segment the metasurface lens into a certain number of concentric rings to facilitate the optimization process.
2. Assigning phase values to unit structures: Assign phase values to individual unit structures within each ring. These phase values act as particles in the optimization process.

3. Calculating the phase of each ring: Compute the overall phase of each ring in a specific direction.
4. Checking phase consistency: Identify the smallest phase difference in the far field as the optimal result.
5. Updating data based on the optimal phase distribution: Use phase values of the optimal phase distribution to update the data (phase values) of all other rings in the lens.
6. Iterative refinement: Iterate the process, recalculating the phase distributions of all rings, selecting a new optimal ring if necessary, and updating the data accordingly.



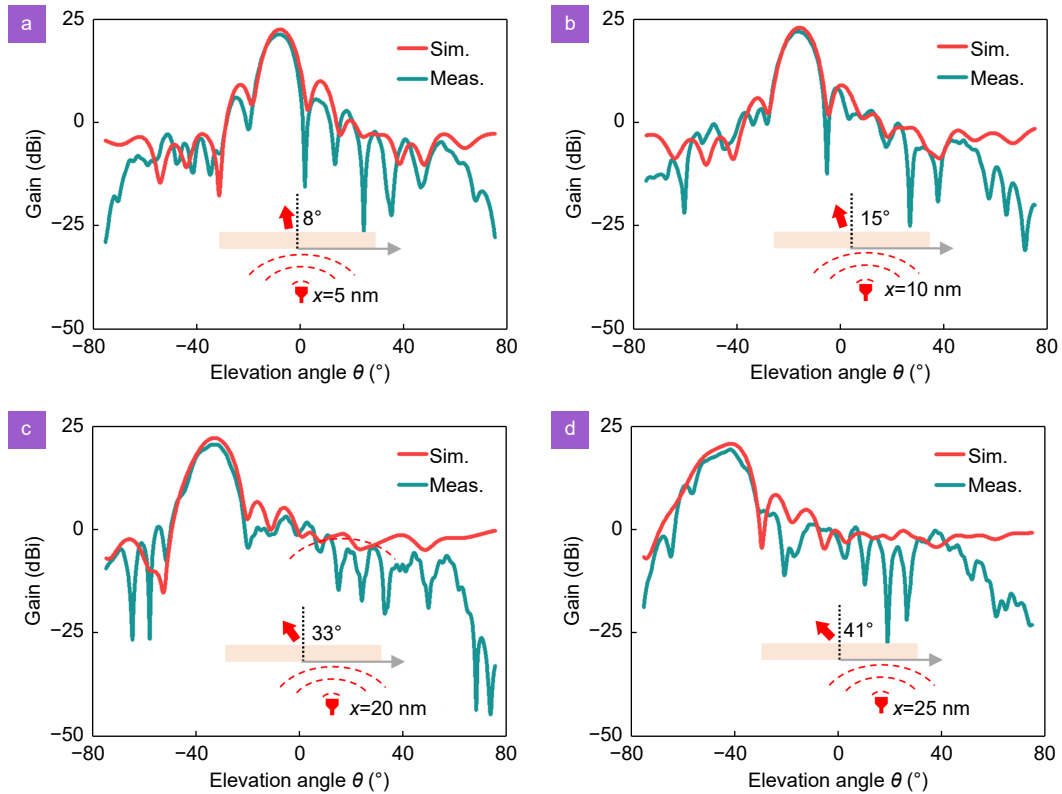
**Fig. S1 | Block diagram of the PA-PSO algorithm and PSO algorithm.** Flow diagrams of (a) the traditional PSO and (b) PA-PSO algorithms. The maximum iteration of both algorithms is set to be 5000 to compare the algorithm convergence speed. The traditional PSO finds the best values based on the radiation intensity while the PA-PSO finds the best values and the weights of the velocities based on the extrema condition as shown in Eq. (7).

Section 3: Feed design for the metalens antenna

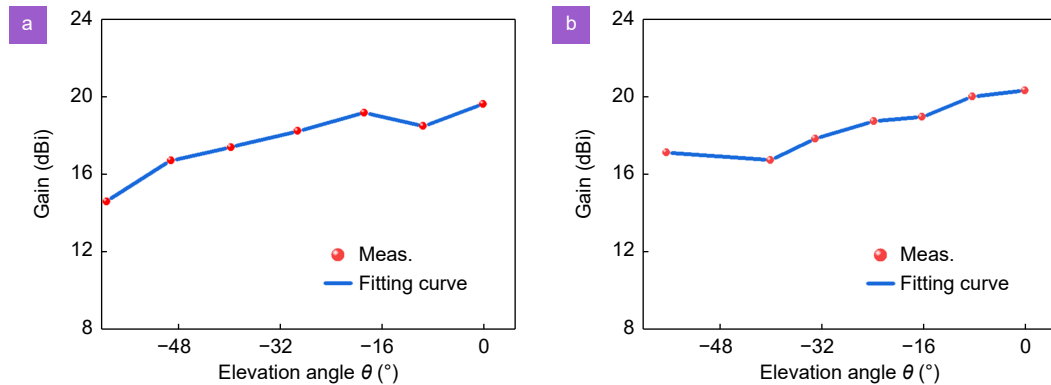


**Fig. S2 | Parameters and performance of the feed antenna.** (a) Schematic diagram and top view of the feed antenna ( $p_1 = 2.53$  mm,  $w_1 = 0.2$  mm,  $a_1 = 1.98$  mm,  $b_1 = 1$  mm,  $c_1 = 1.1$  mm and  $r_1 = 0.19$  mm). (b) S parameter  $S_{11}$  of the feed antenna. (c) Far-field diagram of the feed antenna at 27, 28.5 and 30 GHz. (d) The axial ratio of the feed antenna at 27, 28.5, and 30 GHz.

Section 4: Comparison between experimental and simulation results with different displacements of the feed

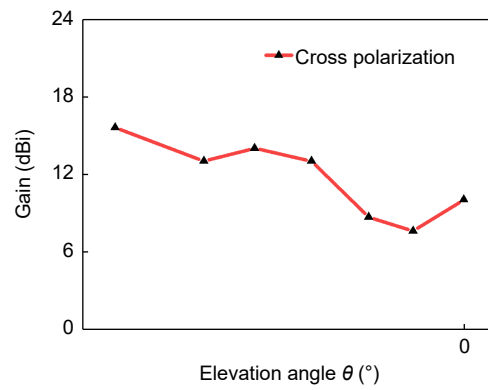


**Fig. S3 | Gain profiles of the metalens antenna when the feed is placed on the focal plane with different displacements  $x$ .** Comparison between experimental and simulation results when the feed displacements are (a) 5 mm, (b) 10 mm, (c) 20 mm and (d) 25 mm.



**Fig. S4 | The relationship between the maximum gain angle and the corresponding gain obtained from testing the feed source at different positions at (a) 27 and (b) 30 GHz.**

The metalens antenna exhibits a broadband response spanning from 27 to 30 GHz. Specifically: at 27 GHz, the antenna provides a gain ranging from 14.5 to 19.5 dBi, with a gain fluctuation within 5 dB. At 28.5 GHz, the gain varies between 18.0 and 21.7 dBi, with a smaller gain fluctuation of 4 dB. At 30 GHz, the antenna offers a gain in the range of 17 to 20.2 dBi, with a similar gain fluctuation of 4 dB. The broadband response is limited by the angular dependence of the feed antenna, which can be further improved by including the feed antenna design in the PA-PSO algorithm.



**Fig. S5 | Gain flatness of cross polarization at 28.5 GHz.**

Within a 15-degree range, there is a polarization isolation exceeding 10 dB. Similarly, within a 41-degree range, the polarization isolation remains at 6 dB. However, as the angle increases beyond these ranges, the polarization isolation gradually decreases to 3 dB.