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Robust measurement of orbital angular momentum of a partially coherent vortex beam under amplitude and phase perturbations

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Section 1: Theoretical construction of a partially coherent Laguerre Gaussian beam with cross-phase using pseudo-mode superposition principle

In the space-frequency domain, the second-order statistical properties of partially coherent sources, propagating along the z-axis, can be described in terms of their cross-spectral density function. In the source plane ($z = 0$), the cross-spectral density function of a partially coherent beam with cross phase has the following form[1]:

$$W(\boldsymbol{r}_1, \boldsymbol{r}_2) = \tau_0(\boldsymbol{r}_1)\,\tau_0^*(\boldsymbol{r}_2)\,\mu(\Delta\boldsymbol{r})\exp\left[\mathrm{i}u\left(x_1y_1 - x_2y_2\right)\right] \ , \tag{S1}$$

where $\boldsymbol{r}_i = (r_{xi}, r_{yi})$ (i=1, 2) denotes the radial coordinates. $\tau_0(\boldsymbol{r})$ denotes a complex function, defined below. $\Delta\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$ is the difference of two position vectors and $\mu(\Delta\boldsymbol{r})$ denotes the degree of coherence function of a partially coherent beam. The last term $\exp[\mathrm{i}u(r_{1x}r_{1y} - r_{2x}r_{2y})]$ is the cross-phase structure, where the quantity u is a measure of the strength of the cross-phase and its sign property (positive or negative value) is only used to determine the rotation direction of the beam[2]. For mathematical convenience and practicality of the conclusions, we consider using the partially coherent Laguerre Gaussian (PCLG) beam, which is the most widely used and easily generated in the laboratory, as the source beam. In this case, the terms $\tau_0(\boldsymbol{r})$ and $\mu(\Delta\boldsymbol{r})$ are given as follows:

$$\tau_0(\boldsymbol{r}) = \left(\frac{\sqrt{2}\boldsymbol{r}}{\omega}\right)^l \exp\left(-\frac{\boldsymbol{r}^2}{\omega^2}\right)\exp\left(\mathrm{i}l\varphi\right) \ , \tag{S2}$$

and

$$\mu(\Delta\boldsymbol{r}) = \exp\left[-\frac{(\Delta\boldsymbol{r})^2}{2\sigma^2}\right] \ , \tag{S3}$$

respectively. φ denotes the azimuthal (angle) coordinates. The quantity l refers to the topological charge. ω and σ denote the transverse beam width and transverse coherence width, respectively.

Consider a physically realizable PCLG beam with a cross-phase, whose cross-spectral density function given by Eq. (S1) can also be represented by the following integral:

$$W(\boldsymbol{r}_1, \boldsymbol{r}_2) = \tau(\boldsymbol{r}_1)\,\tau^*(\boldsymbol{r}_2)\int P(\nu)\exp\left[-\mathrm{i}2\pi\left(\nu\cdot\boldsymbol{r}_2 - \nu\cdot\boldsymbol{r}_1\right)\right]\mathrm{d}^2\nu \ , \tag{S4}$$

with

$$\tau(\boldsymbol{r}) = \tau_0(\boldsymbol{r})\exp\left(\mathrm{i}uxy\right) \ , \tag{S5}$$

and

$$P(\boldsymbol{\nu}) = 2\pi\sigma^2\exp\left(-2\pi^2\sigma^2\boldsymbol{\nu}^2\right) \ . \tag{S6}$$

Next, we employ a pseudo-mode superposition principle[3] to construct the PCLG beam with cross-phase by discretizing Eq. (S4) through Eq. (S6) to yield

$$W(\boldsymbol{r}_1, \boldsymbol{r}_2) = \sum_{m=1}^{M}\sum_{n=1}^{N} P(\boldsymbol{\nu}_{mn})\,\tau(\boldsymbol{\nu}_{mn}, \boldsymbol{r}_1)\,\tau^*(\boldsymbol{\nu}_{mn}, \boldsymbol{r}_2) \ , \tag{S7}$$

where $\boldsymbol{\nu}_{mn} = (\nu_{mnx}, \nu_{mny})$ characterizes the sampling point across the area $\boldsymbol{\nu}$. M and N are the numbers of the sampling points of the function $P(\boldsymbol{\nu})$ in the horizontal and vertical directions, respectively. To achieve high precision, M and N should be large enough. $P(\boldsymbol{\nu}_{mn})$ and $\tau(\boldsymbol{\nu}_{mn}, \boldsymbol{r})$ are the mode weight and discrete pseudo-mode profile, respectively.

To realize the PCLG beam in the laboratory, we must first construct each pseudo-mode, followed by superposing appropriately normalized pseudo-modes. To this end, we can write the electric field of an individual sub-mode as

$$U_{mn}(\boldsymbol{r}) = \sqrt{P(\boldsymbol{\nu}_{mn})}\,\tau(\boldsymbol{\nu}_{mn}, \boldsymbol{r}) \ . \tag{S8}$$

Therefore, through the sub-mode shown in Eq. (S8), the PCLG beam with cross-phase can be constructed by using the superposition method of Eq. (S7).

Section 2: Derivation of the electric field of an individual sub-mode propagating through a ABCD optical system

In this section, we derive the electric field of an individual sub-mode with a cross-phase, propagating in a paraxial

ABCD optical system. The electric field of an individual sub-mode in the source plane is given by Eq. (S8). Within the accuracy of paraxial approximation, the propagation of the electric field of the sub-mode beam at the receiver plane in a paraxial ABCD optical system can be treated by the extended Collins integral:

$$U_{mn}(\boldsymbol{\rho}) = \left(-\frac{\mathrm{i}}{\lambda B}\right) \times \exp(\mathrm{i}kz) \times \iint_{S_1} U_{mn}(\boldsymbol{r}) \cdot \exp\left\{\frac{\mathrm{i}k}{2B}\left[A\left({\rho_x}^2 + {\rho_y}^2\right) + D\left({r_x}^2 + {r_y}^2\right) - 2(\rho_x r_x + \rho_y r_y)\right]\right\} \mathrm{d}r_x \mathrm{d}r_y \,, \quad \text{(S9)}$$

where A, B and D are the elements of the transfer matrix of an optical system, k is the wavenumber, $\boldsymbol{\rho}=(\rho_x, \rho_y)$ denotes the arbitrary position vector in the receiver plane.

Then, on substituting from Eq. (S8) into Eq. (S9), after tedious but straightforward integrating, we obtain the expression for the electric field of an individual sub-mode in the receiver plane

$$\begin{aligned} U_{mn}(\boldsymbol{\rho}) =& \sqrt{P(v_{mn})}\frac{\pi}{\lambda B}\exp(\mathrm{i}kz)(-1)^{j_1+j_2+1} \cdot i^{2j_2+1} \cdot 2^{2j_2-\frac{n+3l}{2}}\left(\frac{2\sqrt{2}}{\omega}\right)^{n-2j_2}\frac{1}{\sqrt{G_1}}\left(\frac{1}{\sqrt{G_2}}\right)^{l-k_0-2j_1-2j_2+1} \\ &\times \sum_{n=0}^{l}\sum_{k_0=0}^{l-n}\sum_{j_1=0}^{[(l-n-k_0)/2]}\sum_{j_2=0}^{[n/2]}\binom{l}{n}\binom{l-n}{k_0}\frac{(l-n-k_0)!}{j_1!(l-n-k_0-2j_1)!}\frac{n!}{j_2!(n-2j_2)!}\left(1-\frac{2}{G_1\omega^2}\right)^{\frac{l-n}{2}}\left(\frac{u}{\sqrt{{G_1}^2\omega^2-2G_1}}\right)^{l-n-k_0-2j_1} \\ &\times \exp\left[\frac{\mathrm{i}kD}{2B}\left({\rho_x}^2+{\rho_y}^2\right)\right]\exp\left[-\left(\frac{k\rho_x-2\pi v_{mnx}B}{2\sqrt{G_1}B}\right)^2\right]\exp\left[\left(\frac{1}{2\sqrt{G_2}}\left(-\frac{\mathrm{i}k\rho_y}{B}+\mathrm{i}2\pi v_{mny}+\frac{(k\rho_x-2\pi v_{mnx}B)u}{2G_1B}\right)\right)^2\right] \\ &\times H_{k_0}\left[\frac{-(\mathrm{i}k\rho_x-\mathrm{i}2\pi v_{mnx}B)}{B\sqrt{{G_1}^2\omega^2-2G_1}}\right]H_{(l-k_0-2j_1-2j_2)}\left[\frac{\mathrm{i}}{2\sqrt{G_2}}\left(-\frac{\mathrm{i}k\rho_y}{B}+\mathrm{i}2\pi v_{mny}+\frac{(k\rho_x-2\pi v_{mnx}B)u}{2G_1B}\right)\right], \end{aligned} \quad \text{(S10)}$$

with

$$G_1 = \frac{1}{\omega^2} - \frac{\mathrm{i}kA}{2B}, G_2 = G_1 + \frac{u^2}{4G_1} \,. \quad \text{(S11)}$$

Based on Eq. (S10), we can numerically study the evolution of individual sub-mode and the PCLG beam with cross-phase, respectively. The intensity of individual sub-mode is $I_{mn}(\rho) = |U_{mn}(\rho)|^2$. The average intensity and cross-spectral density of a PCLG beam with cross-phase on propagation are obtained as:

$$I(\boldsymbol{\rho}) = W(\boldsymbol{\rho}, \boldsymbol{\rho}) = \sum_{m=1}^{M}\sum_{n=1}^{N}|U_{mn}(\boldsymbol{\rho})|^2 \,, \quad \text{(S12)}$$

and

$$W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \sum_{m=1}^{M}\sum_{n=1}^{N}|U_{mn}(\boldsymbol{\rho}_1)U^*_{mn}(\boldsymbol{\rho}_2)| \,. \quad \text{(S13)}$$

Then, we normalize the cross-spectral density function and obtain the degree of coherence function:

$$\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)\Big/\sqrt{W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1)W(\boldsymbol{\rho}_2, \boldsymbol{\rho}_2)} \,. \quad \text{(S14)}$$

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