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Wide-spectrum optical synthetic aperture imaging via spatial intensity interferometry

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Section 1: The spatial field autocorrelation of the point-spread function (PSF)

When imaging in wide spectrum, the point-spread function (PSF) of optical synthetic aperture imaging via spatial intensity interferometry is

$$h_E(\mathbf{u}) = \int_{-\infty}^{\infty} W(k) h_E(\mathbf{u}, k) dk, \quad (S1)$$

where $h_E(\mathbf{u}, k)$ is the point-spread function (PSF) for the wave number $k = 1/\lambda$ (λ is the lambda), $W(k)$ is the spectral distribution of the wide spectrum. Then, the spatial field autocorrelation of h_E is

$$R_{h_E}(\mathbf{u}, \mathbf{u}') = \overline{h_E(\mathbf{u}) h_E^*(\mathbf{u}')} = \iint_{-\infty}^{\infty} W(k) W(k') R_{h_E}(\mathbf{u}, \mathbf{u}'; k, k') dk dk', \quad (S2)$$

where $R_{h_E}(\mathbf{u}, \mathbf{u}'; k, k') = \overline{h_E(\mathbf{u}, k) h_E^*(\mathbf{u}', k')}$.

According to Fresnel diffraction, we have

$$h_E(\mathbf{u}, k) = \int_{-\infty}^{\infty} P(\mathbf{r}, k) \exp[-j2\pi c_0 k (\mathbf{r} - c_1 \mathbf{u})^2] d\mathbf{r}, \quad (S3)$$

where we have omitted a constant term outside the integration in the following derivation to simplify the expression,

$$c_0 = \frac{(z_1 + z_2)}{2z_1 z_2}, c_1 = \frac{z_1}{z_1 + z_2}, \text{ and}$$

$$P(\mathbf{r}, k) = A(\mathbf{r}) S(\mathbf{r}, k), \quad (S4)$$

is the pupil function of optical synthetic aperture imaging via spatial intensity interferometry, $A(\mathbf{r})$ is the aperture function, $S(\mathbf{r}, k) = \exp\{j\varphi(\mathbf{r}, k)\}$ is the modulation phase transfer function, and $\varphi(\mathbf{r}, k)$ is the modulation phase function. Letting $\tilde{\mathbf{r}} = \mathbf{r} - c_1 \mathbf{u}$, we find

$$h_E(\mathbf{u}, k) = \int_{-\infty}^{\infty} P(\tilde{\mathbf{r}} + c_1 \mathbf{u}, k) \exp(-j2\pi c_0 k \tilde{\mathbf{r}}^2) d\tilde{\mathbf{r}}. \quad (S5)$$

Since the ensemble statistics are introduced by the pupil function, interchanging the orders of ensemble statistics and double integration, we obtain

$$\begin{aligned} R_{h_E}(\mathbf{u}, \mathbf{u}'; k, k') &= \iint_{-\infty}^{\infty} d\tilde{\mathbf{r}}_1 d\tilde{\mathbf{r}}_2 \exp[-j2\pi c_0 (k \tilde{\mathbf{r}}_1^2 - k' \tilde{\mathbf{r}}_2^2)] \\ &\quad * \overline{P(\tilde{\mathbf{r}}_1 + c_1 \mathbf{u}, k) P^*(\tilde{\mathbf{r}}_2 + c_1 \mathbf{u}', k')} \\ &\approx \iint_{-\infty}^{\infty} d\tilde{\mathbf{r}}_1 d\tilde{\mathbf{r}}_2 \exp[-j2\pi c_0 k (\tilde{\mathbf{r}}_1^2 - \tilde{\mathbf{r}}_2^2)] \\ &\quad * \overline{P(\tilde{\mathbf{r}}_1 + c_1 \mathbf{u}, k) P^*(\tilde{\mathbf{r}}_2 + c_1 \mathbf{u}', k')}, \end{aligned} \quad (S6)$$

where the approximation is valid for $|\Delta k| = |k - k'| \ll k$. For further simplification, we define

$$R_P(\mathbf{u}, \mathbf{u}'; k, k') \stackrel{\text{def}}{=} \overline{P(\mathbf{u}, k) P^*(\mathbf{u}', k')}. \quad (S7)$$

Assuming the width of the aperture is much greater than the correlation width of the pupil function, we have

$$R_P(\mathbf{u}, \mathbf{u}'; k, k') \approx A(\tilde{\mathbf{u}}) R_s(\mathbf{u}, \mathbf{u}'; k, k'), \quad (S8)$$

with $R_s(\mathbf{u}, \mathbf{u}'; k, k') = \overline{S(\mathbf{u}, k) S^*(\mathbf{u}', k')}$, and $\tilde{\mathbf{u}} = \frac{\mathbf{u} + \mathbf{u}'}{2}$. Making the following definitions:

$$\tilde{\mathbf{r}}_3 = (\tilde{\mathbf{r}}_1 + \tilde{\mathbf{r}}_2)/2, \quad \tilde{\mathbf{r}}_4 = \tilde{\mathbf{r}}_1 - \tilde{\mathbf{r}}_2,$$

to simply the expressions of Eq. (S6) and Eq. (S2), we have

$$R_{h_E}(\mathbf{u}, \mathbf{u}'; k, k') \approx \iint_{-\infty}^{\infty} d\tilde{\mathbf{r}}_3 d\tilde{\mathbf{r}}_4 \exp(-j4\pi c_0 k \tilde{\mathbf{r}}_3 \tilde{\mathbf{r}}_4) A(\tilde{\mathbf{r}}_3 + c_1 \tilde{\mathbf{u}}) R_s\left(\tilde{\mathbf{r}}_3 + \frac{\tilde{\mathbf{r}}_4}{2} + c_1 \mathbf{u}, \tilde{\mathbf{r}}_3 - \frac{\tilde{\mathbf{r}}_4}{2} + c_1 \mathbf{u}'; k, k'\right), \quad (S9)$$

and

$$R_{h_E}(\mathbf{u}, \mathbf{u}') \approx \iiint \int_{-\infty}^{\infty} dk dk' d\tilde{\mathbf{r}}_3 d\tilde{\mathbf{r}}_4 W(k) W(k') A(\tilde{\mathbf{r}}_3 + c_1 \tilde{\mathbf{u}}) \exp(-j4\pi c_0 k \tilde{\mathbf{r}}_3 \tilde{\mathbf{r}}_4) R_s\left(\tilde{\mathbf{r}}_3 + \frac{\tilde{\mathbf{r}}_4}{2} + c_1 \mathbf{u}, \tilde{\mathbf{r}}_3 - \frac{\tilde{\mathbf{r}}_4}{2} + c_1 \mathbf{u}'; k, k'\right), \quad (S10)$$

Equation (S10) shows the relationship of the spatial field autocorrelation function R_{h_E} and the correlation function R_s of modulation phase transfer function.

Section 2: The correlation of modulation phase transfer function

The modulation transfer phase S is assumed to be wide-sense stationary, and its relationship with the surface height η is

$$S(\mathbf{r}, k) = \exp [j2\pi k(n-1)\eta(\mathbf{r})] . \quad (\text{S11})$$

Then, the autocorrelation function of the modulation phase function is given by

$$R_s(\mathbf{r}, \mathbf{r}'; k, k') = \overline{\exp \{j2\pi(n-1)(k\eta(\mathbf{r}) - k'\eta(\mathbf{r}'))\}} . \quad (\text{S12})$$

Considering the characteristic function of the random variable η , the above equation can be written as

$$R_s(\mathbf{r}, \mathbf{r}'; k, k') = M_{\eta(\mathbf{r})} (2\pi(n-1)k, -2\pi(n-1)k') , \quad (\text{S13})$$

here $M_{\eta(\mathbf{r})}$ is the second-order characteristic function of the surface-height function η , which is assumed to be statistically stationary. Assuming η is a Gaussian random variable, in this case, its characteristic function is

$$M_{\eta(\mathbf{r})} (\omega, \omega') = \exp \left\{ -\frac{[(\omega^2 + \omega'^2) \sigma_\eta^2 + 2\omega\omega'R_\eta(\Delta\mathbf{r})]}{2} \right\} , \quad (\text{S14})$$

where σ_η^2 is the height variance, R_η is the height correlation function. Substituting this equation into Eq. (S13), the result is

$$\begin{aligned} R_s(\mathbf{r}, \mathbf{r}'; k, k') &= \exp \left\{ -\frac{[2\pi(n-1)\sigma_\eta]^2}{2} \left[k^2 + k'^2 - \frac{2kk'R_\eta(\Delta\mathbf{r})}{\sigma_\eta^2} \right] \right\} \\ &= \exp \left\{ -\frac{[2\pi(n-1)\sigma_\eta]^2}{2} \left[\Delta k^2 + 2kk' \left(1 - \frac{R_\eta(\Delta\mathbf{r})}{\sigma_\eta^2} \right) \right] \right\} \\ &\approx \exp \left\{ -\frac{[2\pi(n-1)\sigma_\eta]^2}{2} \left[\Delta k^2 + 2k^2 \left(1 - \frac{R_\eta(\Delta\mathbf{r})}{\sigma_\eta^2} \right) \right] \right\} \\ &= R_{sr}(\Delta\mathbf{r}, k) R_{sk}(\Delta k) , \end{aligned} \quad (\text{S15})$$

where

$$R_{sr}(\Delta\mathbf{r}, k) = \exp \left\{ -[2\pi(n-1)\sigma_\eta k]^2 \left[1 - \frac{R_\eta(\Delta\mathbf{r})}{\sigma_\eta^2} \right] \right\} , \quad (\text{S16})$$

and

$$R_{sk}(\Delta k) = \exp \left\{ -\frac{[2\pi(n-1)\sigma_\eta \Delta k]^2}{2} \right\} , \quad (\text{S17})$$

For Gaussian height, the height correlation function is

$$R_\eta(\Delta\mathbf{r}) = \sigma_\eta^2 \exp \left(-\frac{\Delta\mathbf{r}^2}{r_c^2} \right) , \quad (\text{S18})$$

where the transverse correlation length r_c is the radius at which the normalized height correlation falls to $1/e$. So, we obtain the result

$$R_{sr}(\Delta\mathbf{r}, k) = \exp \left\{ -[2\pi(n-1)\sigma_\eta k]^2 \left[1 - \exp \left(-\frac{\Delta\mathbf{r}^2}{r_c^2} \right) \right] \right\} . \quad (\text{S19})$$

Section 3: The spatial intensity autocorrelation of the incoherent intensity impulse response function

Now to get the spatial field autocorrelation function, substitute Eq. (S15) in Eq. (S10):

$$\begin{aligned} R_{IE}(\mathbf{u}, \mathbf{u}') &= \iiint_{-\infty}^{\infty} dk d\Delta k d\tilde{\mathbf{r}}_3 d\tilde{\mathbf{r}}_4 W(k) W(k - \Delta k) A(\tilde{\mathbf{r}}_3 + c_1 \tilde{\mathbf{u}}) \\ &\quad \cdot \exp(-j4\pi c_0 k \tilde{\mathbf{r}}_3 \tilde{\mathbf{r}}_4) R_{sr}(\tilde{\mathbf{r}}_4 + c_1 \Delta\mathbf{u}, k) R_{sk}(\Delta k) \\ &= \iiint_{-\infty}^{\infty} dk d\Delta k d\tilde{\mathbf{r}}_4 W(k) W(k - \Delta k) \mathfrak{F}_a(2c_0 k \tilde{\mathbf{r}}_4) \exp(j4\pi c_0 c_1 k \tilde{\mathbf{u}} \tilde{\mathbf{r}}_4) R_{sr}(\tilde{\mathbf{r}}_4 + c_1 \Delta\mathbf{u}, k) R_{sk}(\Delta k) \\ &= \int_{-\infty}^{\infty} dk W(k) \int_{-\infty}^{\infty} d\Delta k W(k - \Delta k) R_{sk}(\Delta k) \int_{-\infty}^{\infty} d\tilde{\mathbf{r}}_4' \mathfrak{F}_a(\tilde{\mathbf{r}}_4') \tilde{R}_{sr} \left(\tilde{\mathbf{r}}_4' + \frac{k\Delta\mathbf{u}}{z_2}, k \right) , \end{aligned} \quad (\text{S20})$$

where $\tilde{\mathfrak{F}}_a$ is the Fourier transform of the aperture function A , $\Delta k = k - k'$, $\Delta \mathbf{u} = \mathbf{u} - \mathbf{u}'$, and

$$\tilde{R}_{sr}(\Delta \mathbf{r}, k) = \frac{1}{2c_0 k} R_{sr} \left(\frac{\Delta \mathbf{r}}{2c_0 k}, k \right) = \frac{1}{2c_0 k} \exp \left\{ -(2\pi(n-1)\sigma_\eta k)^2 \left[1 - \exp \left(- \left(\frac{\Delta \mathbf{r}}{2c_0 k r_c} \right)^2 \right) \right] \right\}, \quad (\text{S21})$$

and

$$\tilde{\mathfrak{F}}_a(\tilde{\mathbf{r}}) = \tilde{\mathfrak{F}}_a(\tilde{\mathbf{r}}) \exp(j2\pi c_1 \tilde{\mathbf{u}} \tilde{\mathbf{r}}). \quad (\text{S22})$$

Thus, the normalized spatial field autocorrelation of h_E is expressed as

$$\Gamma_{h_E}(\mathbf{u}, \mathbf{u}') = \tau \int_{-\infty}^{\infty} dk W(k) \{W \otimes R_{sk}\}_k \left\{ \tilde{\mathfrak{F}}_a \otimes \tilde{R}_{sr} \right\}_{-\frac{k\Delta \mathbf{u}}{z_2}}, \quad (\text{S23})$$

where τ is the normalized factor.

According to the complex Gaussian moment theorem, the expression of the normalized intensity auto-correlation reduces to

$$\Gamma_{h_I}(\Delta \mathbf{u}) = 1 + |\Gamma_{h_E}(\Delta \mathbf{u})|^2. \quad (\text{S24})$$

Since the spatial fluctuations of h_I are wide-sense stationary, its spatial intensity autocorrelation $G_{h_I}^{(2)}(\Delta \mathbf{u})$ equals to the normalized intensity autocorrelation $\Gamma_{h_I}(\Delta \mathbf{u})$. Substituting Eq. (S23) and Eq. (S24), we find the following result,

$$G_{h_I}^{(2)}(\Delta \mathbf{u}) = 1 + \left| \tau \int_{-\infty}^{\infty} dk W(k) \{W \otimes R_{sk}\}_k \left\{ \tilde{\mathfrak{F}}_a \otimes \tilde{R}_{sr} \right\}_{-\frac{k\Delta \mathbf{u}}{z_2}} \right|^2, \quad (\text{S25})$$

Section 4: The design conditions of sub-aperture spatial random phase modulators

Now, consider Taylor's expansion of R_η in Eq. (S18) up to order one, we have

$$R_\eta(\Delta \mathbf{r}) \approx \sigma_\eta^2 \left(1 - \frac{\Delta \mathbf{r}^2}{r_c^2} \right), \quad (\text{S26})$$

and

$$\tilde{R}_{sr}(\Delta \mathbf{r}, k) \approx \exp \left\{ - \left[\frac{4\pi(n-1)z_1 z_2 \sigma_\eta \Delta \mathbf{r}}{(z_1 + z_2)r_c} \right]^2 \right\} = \tilde{R}_{sr}(\Delta \mathbf{r}) \quad (\text{S27})$$

If the radius of $\tilde{\mathfrak{F}}_a$ is designed much larger than the radius of $\tilde{R}_{sr}$, namely

$$\begin{aligned} \frac{1}{D/2} \gg \frac{(z_1 + z_2)r_c}{4(n-1)z_1 z_2 \sigma_\eta} &\approx \frac{r_c}{4(n-1)z_2 \sigma_\eta} \\ &\Downarrow \\ \text{Condition 1: } D &\ll \frac{8(n-1)z_2 \sigma_\eta}{r_c}, \end{aligned} \quad (\text{S28})$$

where the fact $z_1 \gg z_2$ has been used, then the result of Eq. (S25) is

$$G_{h_I}^{(2)}(\Delta \mathbf{u}) \approx 1 + \left| \tau \int_{-\infty}^{\infty} dk W(k) \{W \otimes R_{sk}\}_k \tilde{\mathfrak{F}}_a \left(-\frac{k\Delta \mathbf{u}}{z_2} \right) \right|^2. \quad (\text{S29})$$

When the radius Δ_w of the spectral distribution W is much smaller than the radius of the R_{sk} , namely

$$\text{Condition 2: } \Delta_w \ll \frac{1}{\sqrt{2\pi}(n-1)\sigma_\eta}, \quad (\text{S30})$$

thus,

$$G_{h_I}^{(2)}(\Delta \mathbf{u}) \approx 1 + \left| \tau \int_{-\infty}^{\infty} dk W(k) R_{sk}(k) \tilde{\mathfrak{F}}_a \left(-\frac{k\Delta \mathbf{u}}{z_2} \right) \right|^2. \quad (\text{S31})$$

To further analysis Eq. (S31), W is given by

$$W(k) = \Gamma_w(k - k_0), \quad (\text{S32})$$

where k_0 is the center wave number. Equation (S31) becomes

$$G_{h_1}^{(2)}(\Delta \mathbf{u}) \approx 1 + \left| \tau \int_{-\infty}^{\infty} dk \Gamma_w(k - k_0) R_{sk}(k) \mathfrak{F}_a \left(-\frac{k \Delta \mathbf{u}}{z_2} \right) \right|^2. \quad (\text{S33})$$

In addition, if Δ_w is assumed much smaller than the radius of $\tilde{\mathfrak{F}}_a \left(-\frac{k \Delta \mathbf{u}}{z_2} \right)$, namely

$$\text{Condition 3: } \Delta_w \ll \frac{2z_2}{DD_t}, \quad (\text{S34})$$

where D_t is the detected width of CCD. Then the function $G_{h_1}^{(2)}(\Delta \mathbf{u})$ can be expressed as

$$G_{h_1}^{(2)}(\Delta \mathbf{u}) \approx 1 + \left| \tau \mathfrak{F}_a \left(-\frac{k_0 \Delta \mathbf{u}}{z_2} \right) \right|^2, \quad (\text{S35})$$

where the condition 1–3 in Eq. (4, 5) is considered.